Model-Free Q-learning with MC and TD [2]

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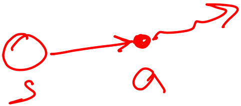
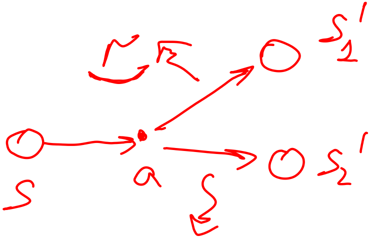
[2] David Silver's Lecture 4

Outline

$$Q(s, a) = \mathbb{E}[G]$$



1. Markov decision processes
2. PI vs VI
3. MC and TD



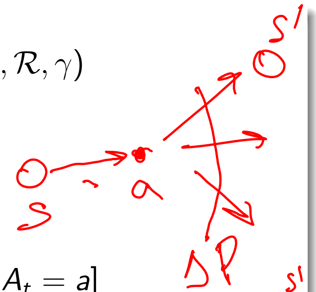
Markov Decision Process

Definition

A Markov Decision Process is a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma)$

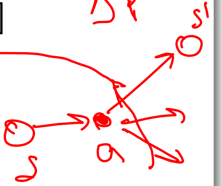
- $\mathcal{S}$ is a (finite) set of states
- $\mathcal{A}$ is a finite set of actions
- $\mathcal{P}$ is a transition probability matix

$$\mathcal{P}^a_{ss'} = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]$$



- $\mathcal{R}$ is a reward function :

$$\mathcal{R}^a_s = \mathbb{E}[R_{t+1} | S_t = s, A_t = a]$$



- γ is a discount factor, $\gamma \in [0, 1]$ Horizon

Recall Policy Iteration

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$

2. Policy Evaluation
   Loop:
   - $\Delta \leftarrow 0$
   - Loop for each $s \in \mathcal{S}$:
     - $v \leftarrow V(s)$
     - $V(s) \leftarrow \sum_{s',r} p(s',r|s,\pi(s))\big[r + \gamma V(s')\big]$
     - $\Delta \leftarrow \max(\Delta, |v - V(s)|)$

   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

fixed $\pi \rightarrow V^{\pi}$

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
   - *old-action* $\leftarrow \pi(s)$
   - $\pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
   - If *old-action* $\neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*

   If *policy-stable*, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2

$V^{\pi} \rightarrow \pi_{i+1}$

Recall : Value Iteration

Value Iteration, for estimating $\pi \approx \pi_*$

Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
| $\Delta \leftarrow 0$
| Loop for each $s \in \mathcal{S}$:
| $v \leftarrow V(s)$
| $V(s) \leftarrow \max_a \sum_{s',r} p(s', r \mid s, a)\big[r + \gamma V(s')\big]$
| $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

Output a deterministic policy, $\pi \approx \pi_*$, such that
  $\pi(s) = \operatorname{argmax}_a \sum_{s',r} p(s', r \mid s, a)\big[r + \gamma V(s')\big]$

Comparison [8]

1. Initialization
   $v(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$

2. Policy Evaluation
   Repeat
   $\Delta \leftarrow 0$
   For each $s \in \mathcal{S}$:
   $temp \leftarrow v(s)$
   $v(s) \leftarrow \sum_{s'} p(s'|s,\pi(s))\left[r(s,\pi(s),s') + \gamma v(s')\right]$
   $\Delta \leftarrow \max(\Delta, |temp - v(s)|)$
   until $\Delta < \theta$ (a small positive number)

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
   $temp \leftarrow \pi(s)$
   $\pi(s) \leftarrow \arg\max_a \sum_{s'} p(s'|s,a)\left[r(s,a,s') + \gamma v(s')\right]$
   If $temp \neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*
   If *policy-stable*, then stop and return v and π; else go to 2

actions are fixed

Figure 4.3: Policy iteration (using iterative policy evaluation) for v_*. This algorithm has a subtle bug, in that it may never terminate if the policy continually switches between two or more policies that are equally good. The bug can be fixed by adding additional flags, but it makes the pseudocode so ugly that it is not worth it. :-)

finding optimal value function

Initialize array v arbitrarily (e.g., $v(s) = 0$ for all $s \in \mathcal{S}^+$)

Repeat
$\Delta \leftarrow 0$
For each $s \in \mathcal{S}$:
$temp \leftarrow v(s)$
$v(s) \leftarrow \max_a \sum_{s'} p(s'|s,a)[r(s,a,s') + \gamma v(s')]$
$\Delta \leftarrow \max(\Delta, |temp - v(s)|)$
until $\Delta < \theta$ (a small positive number)

Output a deterministic policy, π, such that
$\pi(s) = \arg\max_a \sum_{s'} p(s'|s,a)\left[r(s,a,s') + \gamma v(s')\right]$

Figure 4.5: Value iteration.

one policy update (extract policy from the optimal value function

$p(s'|s,a)$

[8] Stackoverflow

Monte-Carlo RL

- MC learns directly from episodes of experience
- Model-free: no knowledge of

$$\mathcal{P}^a_{ss'} \quad or \quad \mathcal{R}$$

- complete episodes
- idea : value = mean return
- Caveat: only episodic MDPS episodes must terminate

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MC Evaluation

$V_\pi(s)$

$Q_\pi(s,a)$

- Goal: learn V_π from episodes under policy π

$$S_1, A_1, R_2, \ldots, S_k \sim \pi$$

- Total discounted reward

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$$

- Value function

$$V_{pi}(s) = \mathbb{E}_\pi[G_t | S_t = s]$$

$$\frac{1}{n} \sum G_t$$

MC Evaluation

- Goal: learn V_π from episodes under policy π

$$S_1, A_1, R_2, ..., S_k \sim \pi$$

- Total discounted reward

$$G_t = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-1} R_T$$

- Value function

$$V_{pi}(s) = \mathbb{E}_\pi[G_t | S_t = s]$$

- Monte-Carlo policy evaluation uses **empirical mean** return instead of expected return

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First-Visit Monte-Carlo Policy Evaluation

s, a, r, s₁, a₁, r₁ ... ~ π

(s   G
s

- To evaluate state s
- The first time-step t that state s is visited in each episode
- $N(s) = N(s) + 1$ # of 1st appearance of s in each game
- $S(s) = S(s) + G_t$ — Σ over many games (episode)
- $\hat{V}(s) = S(s)/N(s)$
- By law of large numbers $V(s) \to V^{\pi}(s)$ as $N(s) \to \infty$

$\mathbb{E}[G_t | s]$

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Incremental Mean

$V^{\pi} \rightarrow \pi$

$$
\begin{aligned}
\mu_k &= \frac{1}{k}\sum_{j=1}^{k} x_j \\
&= \frac{1}{k}\left(\boxed{x_k} + \sum_{j=1}^{k-1} x_j\right) \\
&= \frac{1}{k}\left(x_k + (k-1)\mu_{k-1}\right) \\
&= \underbrace{\mu_{k-1}}_{\text{from } k-1 \text{ games}} + \underbrace{\frac{1}{k}(x_k - \mu_{k-1})}_{\text{diff from last game}}
\end{aligned}
$$

played K games

update policy

$\frac{k-1}{k-1}$

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- Update $V(s)$ incrementally after episode $S_1, A_1, R_2, ..., S_T$
- For each state S_t with return G_t

$$N(S_t) = N(S_t) + 1$$

$$V(S_t) = V(S_t) + \frac{1}{N(S_t)}(G_t - V(S_t))$$

incremental mean $\hat{V}(S_t)$

- In non-stationary problems: running mean

$$V(S_t) = V(S_t) + \alpha(G_t - V(S_t))$$

cum reward for episode for finished game

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Temporal-Difference Learning

don't want to wait until the end of episode

- TD learns from episodes
- model-free $\mathcal{P}^a_{ss'}$ $\mathcal{R}$
- incomplete episodes, bootstrappping diff $\longleftrightarrow$ MC complete episodes
- Update a guess towards a guess idea

MC and TD

$s_0, a_0, r_0, s_1, a_1, r_1 \leftarrow t$

- Goal: learn V_π online from experience under policy π
- Incremental every-visit MC

  predictions

  - $V(S_t) = V(S_t) + \alpha(G_t - \hat{V}(S_t))$
- Simplest TD : TD(0)

  $\approx$

  - Update $V(S_t)$ toward estimated return $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) = V(S_t) + \alpha(\underbrace{R_{t+1} + \gamma V(S_{t+1})}_{\hat{G}_t} - V(S_t))$$

- $R_{t+1} + \gamma V(S_{t+1})$ - TD target
- $\delta_t = R_{t+1} + \gamma V(S_{t+1} - V(S_t)$ - TD error

$V(S_t) = R_{t+1} + \gamma \cdot V(S_{t+1})$

Driving Home Example

		TD	MC
State	**Elapsed Time (minutes)**	**Predicted Time to Go**	**Predicted Total Time**
leaving office	0	30	30
reach car, raining	5	35 ETA	40
exit highway	20	15	35
behind truck	30	10	40
home street	40	3	43
arrive home	43	0	43

G_t

whole traj

MC vs TD

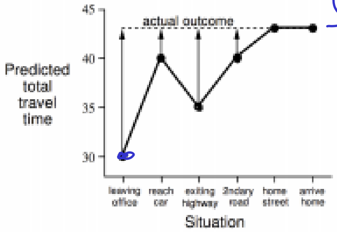
$$R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$

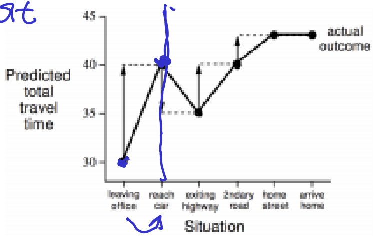
$$V(S_t) = V(S_t) + \alpha\left(G_t - V(S_t)\right)$$

Changes recommended by Monte Carlo methods (α=1)

Changes recommended by TD methods (α=1)

G_t





Advantages and Disadvantages

- TD can learn before knowing the final outcome
  - TD learn after each step
  - MC must end the episode
- TD can learn without the final outcome
  - TD - incomplete sequences
  - TD - continuing envs
  - MC - complete sequences
  - MC - only episodic envs

$G_t = 0$

± 1

t

$R_{t+1} + \underbrace{V(S_{t+1}) - V(S_t)}_{\delta}$

Advantages and Disadvantages (2)

$\sum R \leftarrow$ r.V

MC has high variance, zero bias

- Good convergence
- Not sensitive to initial value
- Simple

$G_t - V(S_t)$

TD(0) $R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$

$1 \quad R + V(S_{t+1})$

$R_{t+1} + \gamma R_{t+2} + \gamma^2 V(S_{t+2}) - V(S_t)$

TD has low variance, some bias

- More efficient than MC
- TD(0) converges to V_π
- (not always for function approximation)
- Sensitive to initial value

TD(λ)

$R + \gamma V(S_{t+1}) - V(S_t)$

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Advantages and Disadvantages (3)

$$p(S_{t+1} | S_t) = p(S_{t+1} | S_t, \cancel{S_{t-1}})$$

TD exploits Markov property

- Usually more efficient in Markov envs

MC does not exploit Markov property

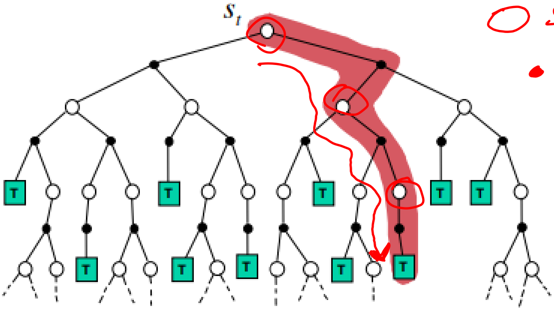
- Usually more effective in non-Markov envs

play the whole game
↓
count

lose ordering

MC Backup

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

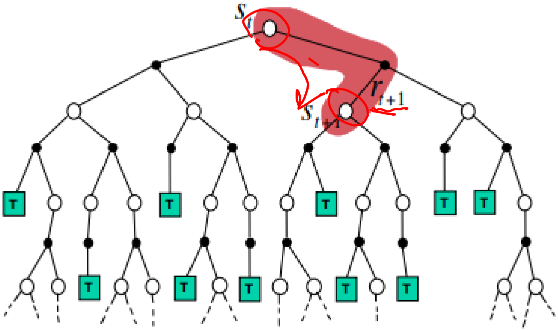


○ states

• actions

TD Backup

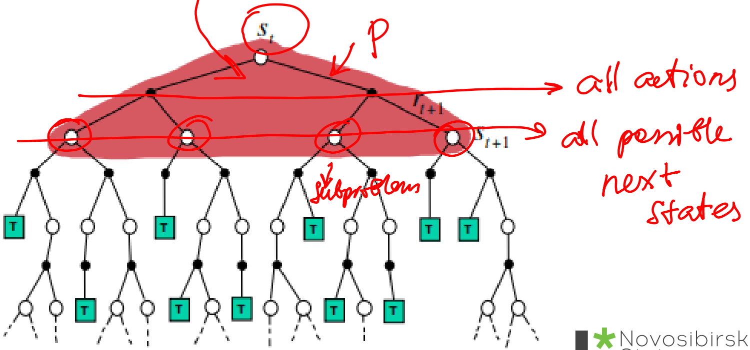
$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$



DP Backup

Bellman Expectation eq.

$$\underbrace{V(S_t)} \leftarrow \mathbb{E}[\underbrace{R_{t+1}} + \gamma \underbrace{V(S_{t+1})}]$$



Bootstrapping and Sampling

Bootstrapping update involves an estimate

- MC
- DP
- TD

Sampling update sample an expectation

- MC
- DP
- TD

Bootstrapping and Sampling

Bootstrapping update involves an estimate

- MC ✗
- DP ✓
- TD ✓

Sampling update sample an expectation

- MC
- DP
- TD

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Bootstrapping and Sampling

Bootstrapping update involves an estimate

- MC ✗
- DP ✓
- TD ✓

Sampling update sample an expectation

- MC ✓
- DP ✗
- TD ✓

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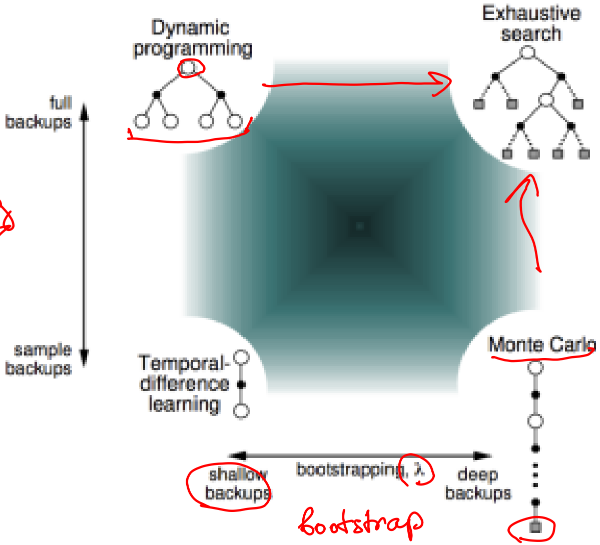
Unified View of RL

Model Free

sampling



brute force

$\mathbb{E} \sim \frac{1}{n} \Sigma$

"sample"

played some games

Value and Policy Iteration Lab

https://bit.ly/2JVv6rc